5. G. Oster, "Density gradients," Sci. Am., 212, No. 12 (1965).
6. L. S. Aĭvazova, T. Ya. Gorbach, K. M. Krolevets, and V. N. Savelov, "Four-element position-sensitive photodiodes" in: Semiconductor Technology and Microelectronics [in Russian], Naukova Dumka, Kiev (1966).
7. D. E. Mowbray, "The use of schlieren and shadowgraph techniques in the study of the flow patterns in density stratified," J. Fluid Mech., 27, No. 3 (1967).
8. A. H. Schooley, "Wake collapse in a stratified fluid," Science, 157, No. 3787 (1967).
9. G. E. Merritt, "Wake growth and collapse in stratified flow," AIAA J., 12, No. 7 (1974).

CURRENT CONTROL WITH A VARIABLE INDUCTANCE

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A number of authors [1-3] point out correctly that the inductance of the commutator circuit, which extracts energy from the accumulator into the load, is "parasitic." Indeed, the voltage on the commutator in the simplest electrotechnical model (Fig. 1) is given by the expression

$$U_b = U_\ell + L_\ell \dot{I}_\ell - MI - L_b \dot{I}_b = V_\ell - M\dot{I} - L_b \dot{I}_b.$$

Here the indices b and ℓ denote quantities referring to the circuit of the circuit breaker and the load; L_b, L_ℓ, and M are the inductance and mutual inductance of the accumulator and load circuits; the dot denotes, as usual, a derivative with respect to the time; U_b and U_ℓ are, in the general case, the nonlinear characteristics of the commutator and the load unrelated with their inductances; and, V_ℓ is the voltage on the load.

For successful operation of the commutator $\dot{I}_b < 0$, $\dot{I} < 0$, whence it follows that $U_b \geq V_\ell$, and in addition the equality obtains at the moment when the switching of the current ends. Further, since the current in the commutator varies from the maximum value to zero and in the load from zero up to the maximum value, it follows from the inequality of the voltages that at the starting stage of switching the power dissipated by the commutator must exceed the power released in the load and in some cases its maximum value also. Thus the inductance in the commutator circuit makes more stringent the conditions of operation of the commutator. For M < 0, which in the case of inductive energy accumulators, can always be made to be satisfied constructionally, the operating conditions of the commutator are eased somewhat. In magnetic cumulation generators this condition cannot always be satisfied.

This is the case when the inductance of the commutator circuit is constant or changes very little. When it changes significantly the picture of the process can be completely different. Switching of the current is possible in this case by virtue of the fact that there arises on the varying inductance an emf ($\varepsilon = -d(LI)/dt$) that permits controlling the current. In explosive magnetic generators a varying inductance is employed to obtain high currents, magnetic fields, and energies [4-6].

The purpose of this work is to show that the current in different electrical circuits can be controlled with the help of both varying and increasing inductances. This method of current control has a number of advantages over, for example, current breakers. They include the absence of dissipation (arcing) of energy in the medium (which destroys its initial electrophysical properties), the possibility of current control according to a presented law, etc.

We shall consider some simple electrotechnical models of circuits employed in pulsed power engineering to obtain high powers. We shall not be concerned with the reasons for the change in the commutating inductance, since they can be diverse and unique to each specific case. In addition, we shall neglect Joule losses, making the assumption that the conductors are perfect.

1. We shall study current control by the commutating inductance in extracting energy from an accumulator into an inductive load for the scheme (Fig. 1) employed by Knopfel [7]

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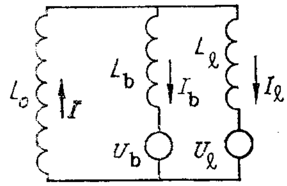

Fig. 1

to demonstrate the operation of an inductive accumulator (U_b and $V_\ell = 0$). In this scheme L_b is the varying inductance, L_0 and L_ℓ are the constant inductances of the accumulator and load, and I, I_b, and I_ℓ are the currents in the accumulator, in the commutating inductance, and in the load. The equations describing the change in the currents in the circuit have the form

$$L_0\dot{I} + (\dot{L_b I_b}) + M\dot{I}_\ell = 0,$$
$$L_\ell \dot{I}_\ell - (\dot{L_b I_b}) + M\dot{I} = 0,\ I = I_b + I_\ell \tag{1.1}$$

with the initial conditions $L_b = L_{0b}$, $I = I_b = I_0$, $I_\ell = 0$. The solution of Eqs. (1.1) will be the currents

$$I_b = I_0\frac{L_{0b}(L_0 + L_\ell + 2M) + L_0L_\ell - M^2}{L_b(L_0 + L_\ell + 2M) + L_0L_\ell - M^2},\quad I_\ell = I_0\frac{(L_b - L_{0b})(L_0 + M)}{(L_0 + L_\ell + 2M)L_b + L_0L_\ell - M^2}.$$

With an increasing commutating inductance ($\dot{L}_b > 0$) the current in it becomes small when $L_b \gg L_0$, L_ℓ and in the limit as $L_b \to \infty$ $I_b \to 0$, while the current in the load approaches its limiting value $I_\ell = I = I_0(L_0 + M)/(L_0 + L_\ell + 2M)$. The energy transferred into the load

$$W_\ell = W_0\frac{(L_0 + M)^2}{(L_0 + L_{0b})}\frac{L_\ell}{(L_0 + L_\ell + 2M)^2}$$

has a maximum at $L_\ell = L_0 + M$, equal to

$$W_{\ell\,\max} = W_0\frac{(L_0 + M)^3}{(L_0 + L_{0b})(2L_0 + 3M)^2}$$

(W_0 is the accumulated energy). This expression agrees with the expression presented in [7] ($W_\ell = (1/4)W_0$) for $L_0 \gg L_{0b}$ and M, which can always be made to be satisfied constructionally in inductive energy accumulators.

We shall study the power developing on the load; this power is equal to $N_\ell = L_\ell I_\ell \dot{I}_\ell$. In the case of a linearly varying commutating inductance ($M \ll L_0$, L_ℓ) it is maximum when

$$L_b = (3/2)L_{0b} + L_0L_\ell/[2(L_0 + L_\ell)]$$

and under the condition of maximum energy transfer ($L_\ell = L_0$) it is equal to

$$N_{\ell\,\max} = \frac{2}{27}\dot{L}_b I_0^2 L_0/(L_0 + 2L_{0b}).$$

One can see from this expression that even for $L_0 \gg L_{0b}$ the maximum power released in the load is an order of magnitude lower than the maximum power in the commutating inductance $N_{b\,\max} \approx \dot{L}_b I_0^2$.

Characteristically the rate of change of the inductance for obtaining maximum power in the inductive load is inversely proportional to the squared accumulated current:

$$\dot{L}_b \approx 13.5 N_{\max}/I_0^2.$$

The maximum voltage on the load for $\dot{L}_b > 0$ occurs initially and is equal to

$$V_{\ell\,\max} = \dot{L}_b I_0 L_\ell L_0/[L_{0b}(L_0 + L_\ell) + L_0L_\ell],$$

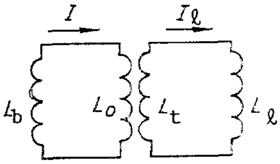

Fig. 2

it is less than the maximum voltage on the commutating inductance ($\sim \dot{L}_b I_0$), and the maximum voltages are equal and are determined by the accumulated current and the rate of change of L_b only for small L_{0b}.

A decreasing commutating inductance results in an increase of the current in the control circuit; in this case ($L_b \to 0$)

$$I_b = I_0 \frac{L_{0b}(L_0 + L_\ell + 2M) + L_\ell L_0 - M^2}{L_0 L_\ell - M^2}, \quad I_\ell = -I_0 \frac{L_{0b}(L_0 + M)}{L_0 L_\ell - M^2}.$$

The energy in the load

$$W_\ell = \frac{1}{2} L_\ell I_\ell^2 = W_0 \frac{L_\ell L_{0b}^2 (L_0 + M)^2}{(L_0 + L_{0b})(L_0 L_\ell - M^2)^2}$$

can be significantly greater than the initial energy W_0 with a strong coupling of the loop, when $L_0 L_\ell \sim M^2$, and with weak coupling, if $L_{0b} \gg L_\ell$ and L_0. In both cases the energy is supplied at the expense of work performed to decrease the inductance L_b. In the first case ($M^2 \sim L_0 L_\ell$) the work is performed in a strong field and in the second case it is performed over a long path.

2. We shall study the transformer scheme for transferring energy into an inductive load (Fig. 2). We shall write the equations describing the change in the current in the form

$$\begin{aligned} (L_b I)^{\cdot} + L_0 \dot{I} + M \dot{I}_\ell &= 0, \\ L_t \dot{I}_\ell + L_\ell \dot{I}_\ell + M \dot{I} &= 0 \end{aligned} \tag{2.1}$$

with the initial conditions $I = I_0$, $I_\ell = 0$, $L_b = L_{0b}$. The notation used in the equations is explained in Fig. 2. Here the coupling of the coils with the transformer is strong, and the interaction of the loops is neglected; then the solution of the system (2.1) are the currents

$$I = I_0 \frac{L_{0b}(L_t + L_\ell) + L_0 L_\ell}{L_b (L_t + L_\ell) + L_0 L_\ell}, \quad I_\ell = I_0 \frac{M(L_b - L_{0b})}{L_b (L_t + L_\ell) + L_0 L_\ell}.$$

The current I and the energy in the accumulator approach zero as $L_b \to \infty$, and the energy in the load approaches the value $W_\ell = W_0 L_\ell M^2/[(L_0 + L_{0b})(L_t + L_\ell)^2]$, which has a maximum at $L_\ell = L_t$ and is equal to $W_{\ell\,max} = (1/4) W_0 M^2/[(L_0 + L_{0b}) L_t]$. For $L_0 \gg L_{0b}$ and $M^2 \approx L_0 L_t$ the energy transferred into the secondary loop is (1/2) W_0.

The power developed in the inductance of the load with a linearly varying inductance L_b has a maximum at $L_b = (3/2) L_{0b} + L_0 L_\ell/[2(L_t + L_\ell)]$ and, if the energy transferred is maximum and $L_0 \gg L_{0b}$ it is equal to $N_{\ell\,max} = (1/16) \dot{L}_b I_0^2$. Like in the first example, it will be more than an order of magnitude lower than the maximum power in the commutator.

The voltage on the load

$$V_\ell = I_0 \dot{L}_b M L_\ell \frac{L_0 L_\ell + L_{0b}(L_t + L_\ell)}{(L_b (L_t + L_\ell) + L_0 L_\ell)^2}$$

is maximum initially, and under the restrictions imposed above on the parameters of the circuit its maximum value $V_{\ell\,max} = \dot{L}_b I_0 \sqrt{L_t/L_0}$ can be much greater for $L_t \gg L_0$ than the maximum value of the voltage $\dot{L}_b I_0$ on the commutating inductance. The maximum value of the current, however, is reached for $L_\ell = 0$ and is equal to $I_0 \sqrt{L_0/L_t}$. For a decreasing commutating in-

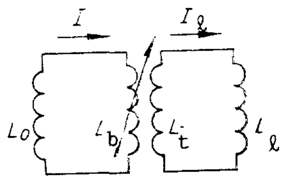

Fig. 3

ductance ($\dot{L}_b < 0$) the maximum power and voltage on the load are reached in the limit $L_b \to 0$, and their limiting values are as follows:

$$N_{\ell\,\max} = I_0^2 \left|\dot{L}_b\right| \frac{L_{0b} L_t}{L_0 L_\ell}\left[1 + \frac{L_{0b}(L_t + L_\ell)}{L_0 L_\ell}\right],$$

$$V_{\ell\,\max} = I_0 \left|\dot{L}_b\right| \sqrt{\frac{L_t}{L_0}}\left[1 + \frac{L_{0b}(L_t + L_\ell)}{L_0 L_\ell}\right].$$

The energy transferred into the load, equal to $L_{0b} \gg L_0$ for $W_\ell = W_0 L_{0b} L_t/(L_0 L_\ell)$, can significantly exceed the initial energy accumulated in the first loop. Formally $W_\ell \to \infty$ as $L_\ell \to 0$. The rapidly increasing value of W_ℓ compared with W_0 is connected with the large amount of work done in reducing L_{0b} in a strong field.

In the examples studied with an increasing inductance of the commutator, if it is assumed that the systems are closed, approximately half of the initially stored energy is transformed into kinetic energy of the moving conductors. For this reason we assume that it is possible to extract energy from the accumulator into the load at the expense of part of the accumulated energy, allowing the commutating inductance to develop in the required direction under the action of electromagnetic forces.

3. Dynamic energy transfer by the transformer scheme can be realized with an arrangement (Fig. 3) in which the commutating inductance is the primary winding of the transformer. We shall represent the equations describing the change in the currents in the system in the form $L_0\dot{I} + (L_b I)^{\cdot} + (M I_\ell)^{\cdot} = 0$, $(L_t + L_\ell)\dot{I}_\ell + (M I)^{\cdot} = 0$ with the initial conditions $I_\ell = 0$, $I = I_0$, $L_b = L_{0b}$ and $M = M_0$; the solution of these equations will be the currents

$$I_\ell = I_0 \frac{M_0(L_0 + L_b) - M(L_0 + L_{0b})}{(L_0 + L_b)(L_t + L_\ell) - M^2}, \quad I = I_0 \frac{(L_t + L_\ell)(L_0 + L_{0b}) - MM_0}{(L_0 + L_b)(L_t + L_\ell) - M^2}.$$

The increase in the inductance L_b can be different. There are cases when k remains constant ($k \approx 1$) as L_b decreases or it decreases ($k \to 0$) or increases from zero to $k \approx 1$.

For $k = 1$ the currents I_ℓ and I vanish when $L_b = L_0^2/L_{0b}$ and $L_b = [(L_0 + L_{0b})^2/L_{0b}](1 + L_\ell/L_t)$, respectively, i.e., they do not vanish simultaneously; for I_ℓ this is observed when $L_{0b} < L_0$. Simultaneous vanishing of these currents results in termination of the process, which is possible when $L_\ell = 0$ and $L_{0b} \ll L_0$. The energy stored in the electromagnetic field, concentrated at first practically completely in L_0, drops to zero. If the system is closed, then the energy initially stored in the first loop is converted completely into the kinetic energy of the moving masses of the increasing inductance. Characteristically the transfer is completed when the inductance L_b reaches its final value, equal to L_0^2/L_{0b}. If the circuit is free ($L_t = 0$), then the transfer of the electromagnetic energy of interaction of the currents into kinetic energy would occur up to the limit $L_b \to \infty$. This difference is due to the significantly larger electromagnetic forces when a transformer is present preventing the current from decreasing as L_b increases. The described situation can be employed for electromagnetic propulsion of bodies.

For the indicated inductances L_b the currents change sign, and as $L_b \to \infty$ the current in the load approaches the limiting value $I_{\ell\ell} = I_0 M_0/L_\ell$ and the energy in the secondary loop approaches $W = W_0 M_0^2(L_t + L_b)/[(L_0 + L_{0b})L_\ell^2]$. The current I, after the sign change, at first increases in absolute magnitude and then decreases, and in the limit $L_b \to \infty$ $I = 0$.

In the case $L_{0b} > L_0$ as L_b increases the character of the change in I remains the same, but now the current I_ℓ does not vanish but rather approaches its limiting value ($I_\ell = I_0 M_0/L_\ell$).

Formally as $L_\ell \to 0$ I_ℓ and W increase without bound. This means that the increase in the commutating inductance occurs when the work on compressing the magnetic field of the current in the transformer is completed. Characteristically the compression of the field is realized not by the current generating the field but rather by the current in the other loop.

In those cases when the coupling constant is a function of L_b it is difficult to analyze the processes. We shall study only the limiting cases. Let $M \to 0$ as $L_b \to \infty$. The current in the primary loop approaches zero in this case, while the current in the secondary loop approaches the value $I_\ell = I_0M_0/(L_t + L_\ell)$ and the energy approaches $W_0M_0^2/[(L_t + L_\ell)(L_0 + L_{0b})]$ This is a small quantity when $L_0 \gg L_{0b}$ and is comparable to W_0 when $L_{0b} \gg L_0$ and $L_\ell \approx 0$.

For $M_0 = 0$ and $k \to 1$ as L_b increases the current I_ℓ increases in absolute magnitude, reaches a maximum at $L_b \approx L_0(L_t + L_\ell)/L_\ell$, and decreases. The maximum value of the current formally increases without bound as $I_\ell \to 0$.

The decrease of the commutating inductance L_b to zero results in the limiting values of the currents $I_\ell = I_0M_0/(L_t + L_\ell)$ and $I = I_0(L_0 + L_{0b})/L_0$. The energy in the system for $L_0 \gg L_{0b}$ is of the order of W_0, and for $L_{0b} \gg L_0$ it can be significantly greater than the initial energy ($\sim L_{0b}/L_0$).

The examples of some circuits, studied above, consisting of ideal conductors suggests that it may be possible to use a varying inductance to control currents in pulsed systems.

LITERATURE CITED

1. L. S. Gerasimov, A. M. Iskol'dskiĭ, Yu. V. Nesterikhin, and V. K. Pikus, "Energy transfer from an inductive accumulator with the help of an electric-explosion current breaker," Zh. Prikl. Mekh. Tekh. Fiz., No. 1 (1975).
2. V. A. Demidov, E. I. Zharinov, S. A. Kazakov, and V. K. Chernyshev, "Extraction of energy from inductive accumulators and explosive magnetic generators into an inductive load with the help of the breaking of a loop," Zh. Prikl. Mekh. Tekh. Fiz., No. 4 (1978).
3. V. A. Demidov, E. I. Zharinov, S. A. Kazakov, and V. K. Chernyshev, "Extraction of energy from explosive magnetic generators into an inductive load with the help of the breaking of a loop," Zh. Prikl. Mekh. Tekh. Fiz., No. 1 (1979).
4. Ya. P. Terletskiĭ, "Production of superstrong magnetic fields by means of rapid compression of conducting shells," Zh. Éksp. Teor. Fiz., 32, No. 2 (1957).
5. E. I. Bichenkov, "Explosive generators," Dokl. Akad. Nauk SSSR, 174, No. 4 (1967).
6. A. D. Sakharov, R. Z. Lyudaev, E. I. Smirnov et al., "Magnetic cumulation," Dokl. Akad. Nauk SSSR No. 1 (1965).
7. G. Knopfel', Superstrong Pulsed Magnetic Fields [Russian translation], Mir, Moscow (1972).